Exam 2 Outline

Calculus of Vector-Valued Functions of a Single Variable: Plane Curves and Space Curves

I. Vector–Valued Functions of One Variable

   A. Be able to parametrize curves described geometrically.

   B. Be able to recognize basic plane and space curves: lines, circles, ellipses, and helices.

II. Limits, Derivatives, and Integrals of Vector–Valued Functions

   A. Be able to compute limits, derivatives, and integrals of vector–valued functions of a single variable.

   B. Understand the connection between plane and space curves and the trajectory of a particle; in particular, know how to compute velocity and acceleration of moving particles.

   C. Know basic differentiation rules especially as they relate to the dot and cross products.

   D. Be able to prove that if $|\vec{r}(t)| = C$ (a constant), then $\vec{r}\,'(t)$ is orthogonal to $\vec{r}(t)$.

III. Projectile Motion

   A. Be able to compute the motion of a point particle launched from the origin at a given velocity in a constant gravitational field.

   B. Be able to answer basic questions about the motion (highest point above the ground, time of flight, total horizontal distance traveled).

IV. Speed and Arc Length

   A. Be able to compute the speed of a parametrized curve, and use it to compute arc lengths of sections of curves.

   B. Be able to compute the arc length function for a given (simple) parametrized curve as measured from a specified point on the curve, and use it to find the arc length parametrization of the curve based at the given point.

V. The TNB-Frame and Curvature

   A. Be able to compute the unit tangent, unit normal, and unit binormal for a given space curve.

   B. Be able to compute the curvature of a given space curve.

VI. Motion in Space

   A. Be able to compute the tangential and normal components of acceleration.

   B. Know the acceleration of a particle moving on a circle at constant speed.

Exam 2 Formula Sheet

- For a regular curve $\vec{r}(t)$,
  - $\hat{T}(t) = \dfrac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|}$
  - $\hat{N}(t) = \dfrac{\hat{T}\,'(t)}{|\hat{T}\,'(t)|}$
  - $\hat{B}(t) = \hat{T}(t) \times \hat{N}(t)$
- For a regular curve $\vec{r}(t)$, the curvature is given by

$$\kappa(t) = \frac{|\hat{T}\,'(t)|}{|\vec{r}\,'(t)|} = \frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|^3}.$$

- For a regular curve $\vec{r}(t)$,

$$\vec{r}\,''(t) = a_{\hat{T}}(t)\hat{T}(t) + a_{\hat{N}}(t)\hat{N}(t)$$

  where

$$a_{\hat{T}}(t) = \frac{\vec{r}\,''(t) \cdot \vec{r}\,'(t)}{|\vec{r}\,'(t)|}$$
$$a_{\hat{N}}(t) = \kappa(t)|\vec{r}\,'(t)|^2 = \sqrt{|\vec{r}\,''(t)|^2 - (a_{\hat{T}}(t))^2}.$$